Lesson 011
Probability Distributions for Discrete Random Variables

Wednesday, October 4

There is no probabilistic difference between modelling a coin flip and a plane crash.

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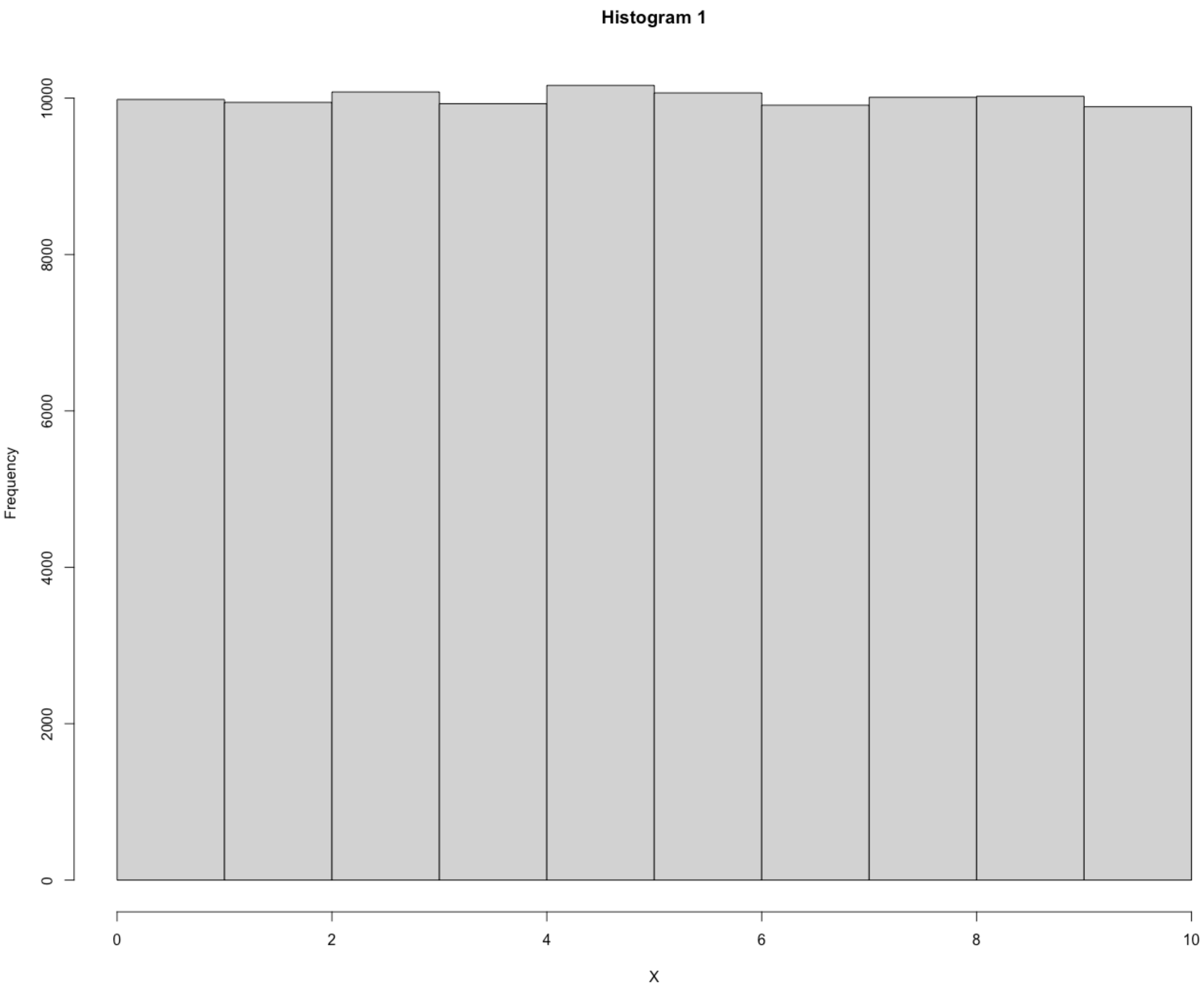
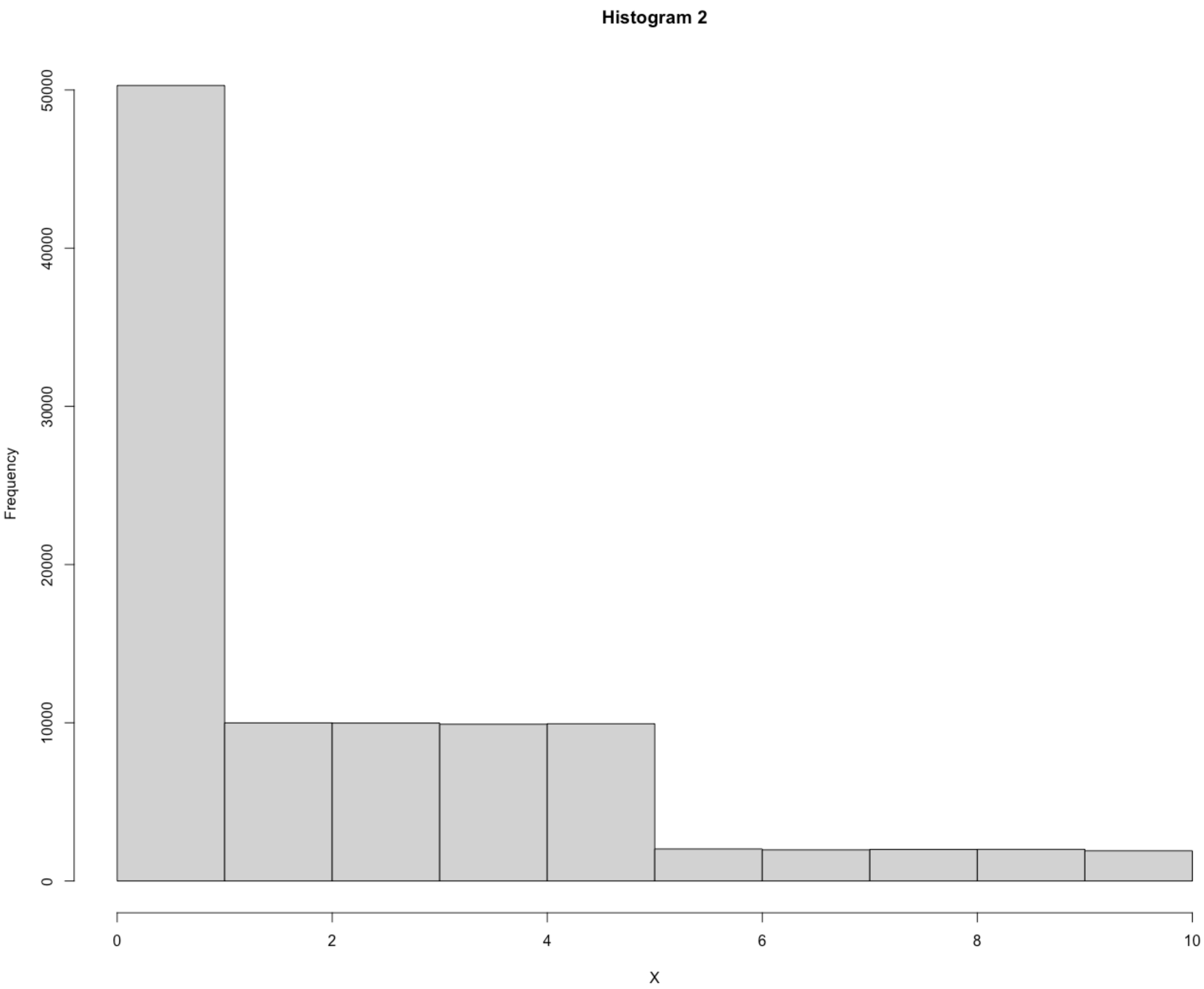

Histogram 1
Frequency
0
2000
4000
6000
8000
10000
0
2
4
6
8
10
X


Histogram 2
Frequency
0
10000
20000
30000
40000
50000
0
2
4
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  - Discrete probability distributions are characterized by **probability mass functions**.

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- We will often write $X \sim p(x)$.

Example

$$p(x) = \begin{cases} 0.5 & x = 0 \\ 0.25 & x = 1 \\ 0.1 & x = 2 \\ 0.15 & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

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$$p(x) = p^x(1-p)^{1-x} \quad x \in \{0,1\}$$

$$p \in (0,1)$$

$$0 \quad \text{otherwise}$$

Consider the following PMF: $p(x) = \begin{cases} 0.5 & x = 0 \\ z & x = 1 \\ 0.125 & x \in \{2, 3\} \\ 0 & \text{otherwise} \end{cases}$. What is the value of z?

$z = 0.5$

0%

$z = 0.375$

0%

$z = 0.25$

0%

$z = 0$

SEE MORE

Consider the following PMF: $p(x) = \begin{cases} 0.5 & x = 0 \\ 0.25 & x = 1 \\ 0.125 & x \in \{2, 3\} \\ 0 & \text{otherwise} \end{cases}$. What is $P(X = \{0\} \cup \{2\})$?

$z = 0.625$

0%

$z = 0.5$

0%

$z = 0.25$

0%

$z = 0.125$

SEE MORE

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- We typically refer to a 1 as a success and a 0 as a failure.

- A biased coin is flipped, which turns up heads 70% of the time.

$$X \sim \text{Bern}(0.7)$$

$$p(x) = (0.7)^x (0.3)^{1-x} \quad x \in \{0,1\}$$

- A commercial plane takes flight. Of all commercial flights, 0.000414% end up crashing.

$$X \sim \text{Bern}(0.99999586)$$

$$p(x) = (0.99999586)^x (0.00000414)^{1-x} \quad x \in \{0,1\}$$

Which of the following random variables is represented by a Bernoulli distribution?

Counting the number of hearts (successes) on 5 draws from a deck of cards, with replacement.

0%

Counting the number of hearts (successes) on 5 draws from a deck of cards, without replacement.

0%

An indicator variable as to whether 5 hearts are observed in 5 draws from a deck of cards, without replacement.

0%

All of the above

0%

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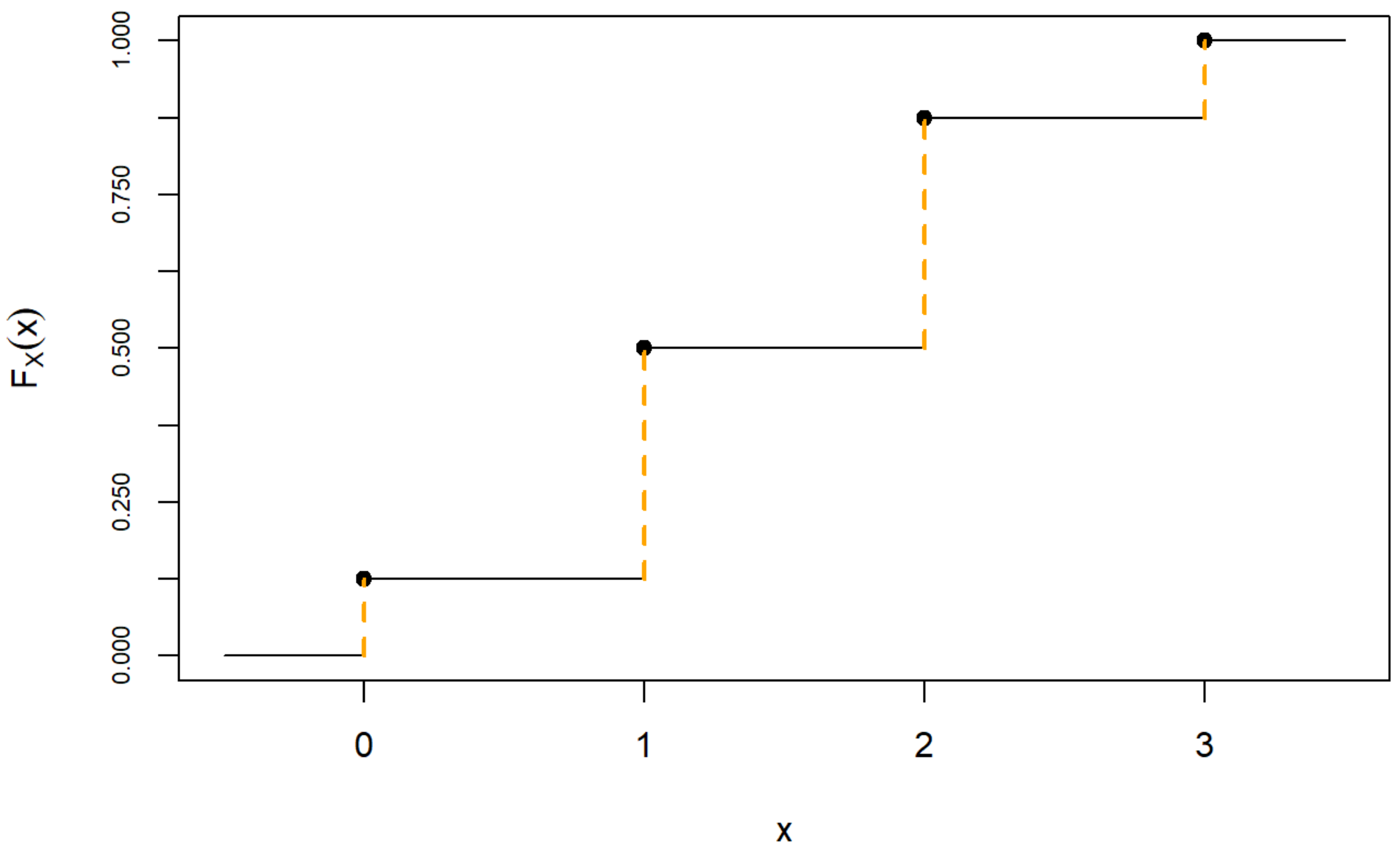
  - For discrete random variables, the CDF will always be a step function.



1.000
0.750
0.500
0.250
0.000
$F_X(x)$
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1
2
3
x

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  - To find the probability of falling in an interval,
    $$P(a < X \leq b) = F(b) - F(a).$$
- It will often be easier to work with a CDF rather than a PMF.

Suppose that $P(Y=8)=0.40$. Moreover, assume that the CDF of Y is given by

$$\begin{cases} 0 & y<1 \\ 0.05 & 1\leq y<2 \\ 0.15 & 2\leq y<4 \\ 0.5 & 4\leq y<8 \\ z & 8\leq y<16 \\ 1 & 16\leq y \end{cases}$$

. What is the value of z?

$z=0.5$

0%

$z=0.75$

0%

SEE MORE

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  - If we right $\lfloor x \rfloor$ as the lowest integer less than or equal to x, then $F_X(x) = 1 - (1-p)^{\lfloor x \rfloor}$.

Which of the following random variables is represented by a Geometric distribution?

Counting the number of draws until a heart is seen in a deck of cards, without replacing them each draw.

0%

Counting the number of draws until a heart is seen in a deck of cards, replacing them each draw.

0%

Counting the number of hearts (successes) in a set number of draws with replacement.

0%

Two or more of the above.

0%